


NPTEL ONLINE CERTIFICATION COURSES

# Management of Fixed Income Securities

Prof. Jitendra Mahakud
Department of Humanities and Social Sciences
Indian Institute of Technology Kharagpur

## Lecture 56: Fixed Income Derivatives-I

# CONCEPTS COVERED

- Forwards and Futures
- T-Bill Futures
- Eurodollar Futures Contract
- T-Bond Futures Contracts
- Forward Rate Agreements
- Basis
- Cost of carry model
- Implied Forward and Futures Rates

NPTEL

# KEYWORDS

- Long position
- Short position
- Trading volume
- Open interest
- Conversion factor
- Hedging
- Cost and carry arbitrage

NPTEL

# Forwards and Futures

- **Forwards**: A forward contract is a customized contract between two entities, where settlement takes place on a specific date in the future at today's pre-agreed price.
- **Futures**: A futures contract is an agreement between two parties to buy or sell an asset at a certain time in the future at a certain price. Futures contracts are special types of forward contracts in the sense that the former are standardized exchange-traded contracts

NPTEL

## Some Concepts

- Buyer is said to have a **long position** and seller has a **short position**
- The act of buying is called **going long** and the act of selling is called **going short**
- When one trader buys and other sells a forward contract, the transaction generates on contract of **trading volume**
- At any moment in time, there is some number of future contracts obligated for delivery and this number is called **open interest**
- **Settlement price**: the price just before the final bell each day (used for the daily settlement process)

NPTEL

# Forward Contracts vs Futures Contracts

| FORWARDS | FUTURES |
| --- | --- |
| Private contract between 2 parties | Exchange traded |
| Not standardized | Standard contract |
| Usually 1 specified delivery date | Range of delivery dates |
| Settled at maturity | Settled daily |
| Delivery or final cash settlement usually occurs | Contract usually closed out prior to maturity |
| Some credit risk | Virtually no credit risk |

NPTEL

# T-Bill Futures

- T-bill futures contracts call for the delivery (short position) or purchase (long position) of a T-bill with a maturity of 91days and a face value (F) of $1 million
- Futures prices on T-bill contracts are quoted in terms of an index. This index, I, is equal to 100 minus the annual percentage discount rate, $R_D$, for a 90-day T-bill

$$I = 100 - R_D(\%)$$

- Given a quoted index value or discount yield, the actual contract price on the T-bill futures contract is:

$$f_0 = \frac{100 - R_D\%(90/360)}{100} 1000000$$

NPTEL

# T-Bill Futures

If a T-bill futures contract quoted at a settlement index value of 95.62, Face Value: $1 Million, Find the futures contract price and implied YTM.

$R_D$ = (100-95.62= 4.38%)

Futures Price: $$f_0 = \frac{100 - 4.38(90/360)}{100} 1000000 = 989050$$

$$YTM_f = \left[\frac{F}{f_0}\right]^{365/91} - 1$$

$$YTM_f = \left[\frac{1000000}{989050}\right]^{365/91} - 1 = 0.04515$$

NPTEL

# Eurodollar Futures Contract

- A Eurodollar is a dollar deposited in a bank outside the United States
- A Eurodollar interest rate is the rate that one large international bank is willing to lend to another large international bank.
- The LIBOR is used as a benchmark rate on bank loans and deposits.
- The major difference between the Eurodollar and T-bill contracts is that Eurodollar contracts have **cash settlements** at delivery, while T-bill contracts call for the actual delivery of the instrument.
- When a Eurodollar futures contract expires, the cash settlement is determined by the futures price and the settlement price.
- The settlement price or expiration futures index price is 100 minus the average three-month LIBOR offered by a sample of designated Euro-banks on the expiration date:

$$\text{Expiration Futures Price} = 100 - \text{LIBOR}$$

NPTEL

## Eurodollar Futures Contract

- Eurodollar futures are quoted in terms of an index equal to 100 minus the annual discount rate, with the actual contract price found by using the following equation:

$$f_0 = \frac{100 - R_D\%(90/360)}{100}\ \$1000000$$

- Given a settlement index value of 95.09 on a Eurodollar contract, the actual futures price would be:

$$f_0 = \frac{100 - 4.91(90/360)}{100}\ \$1000000 = \$987725$$

NPTEL

# T-Bond Futures Contracts

- T-bond futures prices are quoted in dollars and 32nds for T-bonds with a face value of $100.
- If the quoted price on a T-bond futures were of 109-14 (i.e., 109 14/32 or 109.437), the price would be $109437 for a face value of $100000.
- The contract calls for the delivery or purchase of a T-bond with a maturity of at least 15 year.
- The futures contract is based on the delivery of a T-bond with a face value of $100000.
- Delivery can occur at any time during the delivery month
- A conversion factor used to determine the actual price of the deliverable bond

NPTEL

# T-Bond Futures Contracts

- The actual price paid on the T-bond or revenue received by the seller in delivering the bond on the contract is equal to the quoted futures price times the **conversion factor** (CF), on the delivered bond plus any accrued interest:

$$\text{Seller's Revenue} = (\text{Quoted Futures Price})(\text{CF}) + \text{Accrued Interest}$$

**Example***:* At the time of delivery, if the delivered bond has a CF of 1.3 and accrued interest of $2 and the quoted futures price is 94-08, then the cash received by the seller of the bond and paid by the futures purchaser would be $= (94.25)(1.3) + 2 = 124.525$

- T-note contracts are similar to T-bond contracts, except that they call for the delivery of any T-note with maturities between 6 1/2 and 10 years

NPTEL

# Forward Rate Agreements

- Forward rate agreement is an OTC contract designed to fix the interest rate hat will apply to either borrowing or lending a certain principal amount during a specified future time period
- When the FRA is first negotiated, it has zero value as specified interest rate is equal to forward rate
- FRAs are based on LIBOR. The trader who will borrow certain principal amount at LIBOR for a future period can enter into an FRA where for the specific time period, LIBOR will be received and predetermined fixed rate will be paid on the principal amount. The trade who will earn interest at LIBOR for future can take reverse position
- Usually the contract is settled at the beginning of the period by payment of the present value of the payoff

NPTEL

# Forward Rate Agreements

- Suppose company A has agreed to lend money at LIBOR to company B for the period of time between $T_1$ and $T_2$ and enters an FRA to fix the rate of interest it will receive.
- Let $R_K$= The rate of interest agreed to in FRA, RF= Forward LIBOR rate for the period between times $T_1$ and $T_2$ calculated today, $R_M$= Actual LIBOR rate observed in the market at times $T_1$ for the period between times $T_1$ and $T_2$, L= Principal underlying the contract. We assume that the rates $R_K$, $R_F$ and $R_M$ are all measured with a compounding frequency reflecting the length of the period (e.g. if $T_2$-$T_1$=0.5, they are expressed in semi-annual compounding)
- Company A would earn $R_M$ from LIBOR loan, the FRA implies that it will earn $R_K$. The extra interest rate it will earn: $R_K$-$R_M$

NPTEL

# Forward Rate Agreements

- The cash flow to company A at time $T_2$: $L (R_K-R_M)(T_2-T_1)$
- Company B's cash flow at time $T_2$: $L (R_M-R_K)(T_2-T_1)$
- FRAs are usually settled in time T1.
- For company A, the cash flow is the present value of time T1 of : $L (R_K-R_M)(T_2-T_1)$ received at time T2.
- For company B, the pay-off is the present value of time T1 of : $L (R_M-R_K)(T_2-T_1)$ received at time T2.
- The discount rate is the risk free rate

NPTEL

# Futures Positions

- A futures holder can take one of two positions on a futures contract: a long position (or futures purchase) or a short position (futures sale).
- In a long futures position, the holder agrees to buy the contract's underlying asset at a specified price, with the payment and delivery to occur on the expiration date (also referred to as the delivery date).
- In a short futures position, the holder agrees to sell an asset at a specific price, with delivery and payment occurring at expiration.

NPTEL

# Futures Hedging

- In a long hedge (or hedge purchase), a hedger takes a long position in a futures contract to protect against an increase in the price of the underlying asset or commodity.
- Long hedge positions on debt securities are used by money-market managers, fixed-income managers, and dealers to lock in their costs on future security purchases.
- In a short hedge, a hedger takes a short futures position to protect against a decrease in the price of the underlying asset.
- Short hedge positions are used (i) By bond and money market managers, investment bankers, and dealers who are planning to sell securities in the future, (ii) By banks and other intermediaries to lock in the rates they pay on future deposits, (iii) By corporates and other borrowers who want to lock in the future rates on their loans or who want to fix the rates on the floating rate loans.

NPTEL

# Basis

- The difference between the future (or forward) price and the spot price is called the basis
- For most futures (and forward) contracts, the futures price exceeds the spot price before expiration and approaches the spot price as expiration nears.
- The basis usually is positive and decreasing over time, equaling or nearing zero at expiration
- Futures and spot prices also tend to be highly correlated with each other, increasing and decreasing together; their correlation, though, is not perfect.

NPTEL

# Cost of carry model

- The relationship between the spot price and the futures or forward price can be explained by the cost of carry model or carrying cost model.
- In this model, arbitrageurs ensure that the equilibrium forward price is equal to the net costs of carrying the underlying asset to expiration.
- In terms of the carrying-cost model, the price difference between futures and spot prices can be explained by the costs and benefits of carrying the underlying asset to expiration.
- For futures on securities, the carrying costs include the financing costs of holding the underlying asset to expiration and the benefits include the coupon interest or dividends earned from holding the security.

NPTEL

# Carrying-Cost Model (Pricing of a T-Bill Contract)

- With no coupon interest, the underlying T-bill does not generate any benefits during the holding period and the financing costs are the only carrying costs.

$$f_0 = S_0 (1+R_f)^T$$

$f_0$ = contract price on the T-bill futures contract.

T = time to expiration on the futures contract.

$S_0$ = current spot price on a T-bill identical to the T-bill underlying the futures

$R_f$ = risk-free rate or repo rate

$S_0(1+R_f)^T$ = financing costs of holding a spot T-bill

NPTEL

# Carrying-Cost Model (Pricing of a T-Bill Contract)

- If the rate on a 161-day spot T-bill is 5.7% and the risk-free (RF) rate for 70 days is 6.38%, then the price on a T-bill futures contract with an expiration of 70 days would be:

$$f_0 = S_0 (1 + R_f)^T$$

$$f_0 = 97.5844(1.0638)^{70/365} = 98.74875$$

where:

$$S_0 = \frac{100}{(1.057)^{161/365}} = 97.5844$$

If the market price does not equal equilibrium price then arbitrageurs would take a position in the futures and an opposite position in the spot. This arbitrage strategy is referred to as a ***cash-and-carry arbitrage.***

NPTEL

# Carrying-Cost Model (Pricing of a T-Bill Contract)

Let $f^M$ = Market price =Rs. 99

Initial Position

$$f_0^M > f_0 = S_0(1+R_f)^T$$

$$99 > 98.74875 = 97.5844(1.0638)^{70/365}$$

**Short Futures:**
Agree to sell 91-day T-bill 70 days from now

**Long Spot:**
Borrow 97.5844 at 6.38% for 70 days and Buy 161-day T-bill

Expiration

- Sell the T-bill (which now would have a maturity of 91 days) on the futures for 99
- Pay financing debt of 98.74875:

  $97.5844(1.0638)^{70/365} = 98.74875$
- Cash flow = Rs. 0.25125 per Rs. 100 Face Value

NPTEL

# Carrying-Cost Model (Pricing of a T-Bill Contract)

Let $f^M$ = Market price =Rs. 98

**Initial Position**

$$f_0^M < f_0 = S_0(1+R_f)^T$$

$$98 < 98.74875 = 97.5844(1.0638)^{70/365}$$

**Long Futures:**
- Agree to buy 91-day T- bill 70 days from now for 98

**Short Spot:**
- Borrow the 161-day T-bill
- Sell it for 97.5844
- Invest the proceeds at 6.38% for 70 days

**Expiration**
- 70 days later (expiration), buy the bill (which has a maturity of 91 days) on the futures for 98
- Use the bill to close short T-bill position
- Collect 98.74875 from investment: $97.5844(1.0638)^{70/365} = 98.74875$
- Cash flow = Rs 0.74875 per Rs. 100 Face Value

NPTEL

# Implied and Actual Repo Rate

- The implied repo rate is defined as the rate in which the arbitrage profit from implementing the cash-and-carry arbitrage strategy is zero

$$\pi = f_0 - S_0(1+R_f)^T$$

$$0 = f_0 - S_0(1+R_f)^T$$

$$R = \left[\frac{f_0}{S_0}\right]^{1/T} - 1$$

- The actual repo rate is the rate used in solving for the equilibrium futures price in the carrying-cost model

NPTEL

# Implied and Actual Repo Rate

- Buy the 161-day T-bill at $S_0^{161} = 97.5844$
- Lock in selling price 70 days later by going short in the 70-day T-bill futures at $f_0 = 98.74875$
- Rate on investment = Implied Repo Rate = 6.38% = Actual Repo Rate

$$R_{syn} = \left[\frac{98.74875}{97.5844}\right]^{365/70} - 1 = 0.0638$$

NPTEL

# Implied Forward and Futures Rates

- The other condition implied by the carrying-cost model is the equality between the rate implied by the futures contract, ($YTM_f$) and the implied forward rate ($F_r$)
- Implied Futures Rate = Implied Forward Rate

$$YTM_f = F_R$$

$$\left[\frac{F}{f_0}\right]^{365/91} - 1 = \left[\frac{S(T)}{S(T+91)}\right]^{365/91} - 1$$

where:

F = face value on the spot T-bill

T + 91 = maturity of the spot T-bill

NPTEL

# Implied Forward and Futures Rates

- The ***right-hand side*** of the equation is the *implied forward rate.* This rate is determined by the current spot prices on T-bills maturing at T and at T + 91

$$F_R = \left[\frac{S(70)}{S(161)}\right]^{365/91} - 1 = \left[\frac{98.821}{97.5844}\right]^{365/91} - 1 = 0.0518$$

- The ***left-hand side*** of the equation is the *rate implied on the futures contract*. If an investor purchases a 91-day T-bill on the futures contract at the equilibrium price, then the implied futures rate will be equal to the implied forward rate.

$$YTM_f = \left[\frac{F}{f_0}\right]^{365/91} - 1 = \left[\frac{100}{98.74875}\right]^{365/91} - 1 = 0.0518$$

NPTEL

# Implied Forward and Futures Rates

- The implied forward rate is the interest rate attained at a future date that is implied by current rates.
- This rate can be attained by a **locking-in strategy** consisting of a short position in a shorter term bond and a long position in a longer term one.
- In terms of our example, the implied forward rate on a 91-day T-bill investment to be made 70 days from the present, $F_r(91,70)$, is obtained by:
  - Selling short the 70-day T-bill at 98.821 (or equivalently borrowing 98.821 at 6.38%)
  - Buying $S_0(T)/S_0(T+91) = S_0(70)/S_0(161) = 98.821/97.5844 = 1.01267$ issues of the 161-day T-bill
  - Paying 100 at the end of 70 days to cover the short position on the maturing bond (or the loan)
  - Collecting 1.01267(100) at the end of 161 days from the long position
- This locking-in strategy would earn an investor a return of Rs. 101.267, 91 days after the investor expends Rs. 100 to cover the short sale; thus, the implied forward rate on a 91-day investment made 70 days from the present is

$$F_R(91,70) = \left[\frac{\$101.267}{\$100}\right]^{365/91} - 1 = 0.0518$$

NPTEL

# CONCLUSIONS

- A forward contract is a customized contract between two entities, where settlement takes place on a specific date in the future at today's pre-agreed price and future contracts are specialized forward contracts, which are standardized
- Major fixed income derivatives include T-bill futures, Eurodollar futures and T-Bond futures
- Forward rate agreement is an OTC contract designed to fix the interest rate hat will apply to either borrowing or lending a certain principal amount during a specified future time period
- A futures holder can take one of two positions on a futures contract: a long position (or futures purchase) or a short position (futures sale).
- The difference between the future (or forward) price and the spot price is called the basis
- The relationship between the spot price and the futures or forward price can be explained by the cost of carry model or carrying cost model.

NPTEL

# REFERENCES


- Johnson, S. R (2010): Bond Evaluation, Selection and Management, John Wiley & Sons, 2nd Edition.
- Fabozzi, J. Frank and Mann, V. Steven (2005): The Hand Book of Fixed Income Securities, Tata McGraw-Hill, 7th Edition.
- Hull, C. John and Basu Sankarshan (2018): Options, Futures and Other Derivatives, Pearson Education, 10th Edition.


NPTEL




NPTEL ONLINE CERTIFICATION COURSES

# Management of Fixed Income Securities

Prof. Jitendra Mahakud
Department of Humanities and Social Sciences
Indian Institute of Technology Kharagpur

## Lecture 57: Fixed Income Securities Derivatives-II

# CONCEPTS COVERED

- Options
- Moneyness
- Futures options
- T-Bond Futures Options
- Eurodollar and T-Bill Futures Options
- OTC derivatives

NPTEL

# KEYWORDS

- Call option
- Put option
- American option
- European option
- In the money
- Out of the money
- At the money
- Interest rate call
- Interest rate put
- Cap
- Floor

NPTEL

# Options

- An option is a security that gives the holder the right (but not the obligation) to buy (sell) an asset at a specific price (exercise price) on or possibly before a specific date (expiration date).
- A **Call** option is an option that gives the holder the right to buy a specific asset or security.
- A **Put** option is an option that gives the holder the right to sell a specific asset or security.
- **American Option**: Option that can be exercised at any time on or before the expiration date.
- **European Option**: Option that can be exercised only on the expiration date.

NPTEL

# Concept of Moneyness

| Condition | Call Option | Put Option |
|---|---|---|
| $S_0 > E$ | In-the-Money | Out-of-the Money |
| $S_0 < E$ | Out-of-the Money | In-the-Money |
| $S_0 = E$ | At-the-Money | At-the-Money |

NPTEL

## Futures Options

- A futures option gives the holder the right to take a position in a futures contract.
- **Call option on a futures contract** gives the holder the right to take a long position in the underlying futures contract.
- **Put option on a futures option** gives the holder the right to take a short position in the underlying futures contract.
- Spot options and futures options are equivalent if
  - The options and the futures contracts expire at the same time
  - The carrying-costs model holds
  - The options are European

NPTEL

# T-Bond Futures Options

- The premiums on the call and put option contracts on the T-bond and T-note futures are quoted as a percentage of the face value of the underlying bond or note.
- A buyer of the June 109 T-bond futures call trading at 2 –60 (or 2 60/64 = 2.9375 per Rs. 100 face value) would pay Rs. 2,937.50 for the option to take a long position in the June T-bond futures at an exercise price of Rs. 109,000.
- If long-term rates were to subsequently drop, causing the June T-bond futures price to increase to $f_t$ = 115, then the holder, upon exercising, would have a long position in the June T-bond futures contract, the margin and profit of the contract would be:

$$Margin Value = \frac{f_t - X}{100}(100000) = \left[\frac{115-109}{100}\right]100000 = 6000$$

$$\pi = Rs.6000 - Rs.2937.50 = Rs.3062.50$$

- By contrast, if long-term rates were to stay the same or increase, then the call would be worthless and the holder would simply allow it to expire, losing the Rs.2937.50 premium.

NPTEL

# Eurodollar and T-Bill Futures Options

- The exercise prices on the Eurodollar and T-bill contracts are quoted in terms of an index (I) equal to 100 minus the annual discount yield: $I = 100 - R_D$.
- The option premiums are quoted in terms of an index point system.
- For T-bill and Eurodollars, the dollar value of an option quote is based on a $25 value for each basis point underlying a $1 million T-bill or Eurodollar.
- The actual quotes are in percents; thus a 1.25 quote would imply a price of $25 times 12.5 basis points: ($25)(12.5) = $312.50.
- For the closest maturing month, the options are quoted to the nearest quarter of a basis point
- For other months, they are quoted to the nearest half of a basis point.

NPTEL

# Eurodollar and T-Bill Futures Options

The actual price on a May Eurodollar call with an exercise price of 93.5 quoted at 6.92 is:

The price is obtained by (i) Rounding the 6.92 quoted price to 6.925, (ii) Converting the quote to basis points (multiply by 10) and (iii) Multiplying by $25: (6.925)(10)($25) = $1731.25.

An investor buying the 93.5 May call would therefore pay $1731.25 for the right to take a long position in the $1 million May Eurodollar futures contract at an exercise price of:

$$X = \frac{100 - R_D(90/360)}{100}\$1{,}000{,}000$$

$$X = \frac{100 - (100 - 93.5)(90/360)}{100}\$1{,}000{,}000 = \$983750$$

If short-term rates were to subsequently drop, causing the May Eurodollar futures price to increase to an index price of 95.5 ($R_D$ = 4.5 and $f_t$ = [[100 – 4.5(90/360)]/100]($1,000,000) = $988,750), the holder, upon exercising, would have a long position in the May Eurodollar futures contract and a futures margin account worth $5000.

# Eurodollar and T-Bill Futures Options

If she closed the position at 95.5, the investor would realize a profit of :

$$Margin\ Value = f_t - X = \$988750 - \$983750 = \$5000$$

$$Margin\ Value = \$25(Futures\ Index - Exercsie\ Index)$$

$$Margin\ Value = \$25[95.5 - 93.5](100) = \$5000$$

$$\pi = \$5000 - \$1731.25 = \$3268.75$$

If short-term rates were at $R_D$ = 5.5% and stayed there or increased, then the call would be worthless and the holder would simply allow the option to expire, losing her $1731.25 premium.

NPTEL

# OTC T-Bond and T-Note Options

- OTC dealers often offer or will negotiate contracts giving the holder to right to purchase or sell a *specific* T-bond or T-note.
- For example, a dealer might offer a T-bond call option to a fixed income manager giving him the right to buy a specific T-bond maturing in year 2022 and paying a 7% coupon with a face value of $100,000.
- Because the option contract specifies a particular underlying bond, the maturity of the bond, as well as its value will be changing during the option's expiration period.
- The underlying bond or note can pay coupon interest during the option period.
- If the option holder exercises on a non-coupon paying date, the accrued interest on the underlying bond must be accounted for.
- For a T-bond or T-note option, this is done by including the accrued interest as part of the exercise price.

NPTEL

# OTC T-Bond and T-Note Options

- Like futures options, the exercise prices on a spot T-bond or T-note option is quoted as an index equal to a proportion of a bond with a face value of $100.
- If the underlying bond or note has a face value of $100,000, then the exercise price would be:

$$X = \left[\frac{\text{Index}}{100}\right](\$100{,}000) + \text{Accrued Interest}$$

- The prices of spot T-bond and T-note options are typically quoted like futures T-bond options in terms of points and 32nds of a point.
- The price of a call option on a $100,000 T-bond quoted at 1 8/32 is
- (1.25/100)($100,000) = $1250

NPTEL

# Interest Rate Call

- In addition to option contracts on specific securities, the OTC market also offers a number of interest-rate option products like interest rate call and the interest rate put.
- An interest rate call, also called a caplet, gives the buyer a payoff on a specified payoff date if a designated interest rate, such as the LIBOR, rises above a certain exercise rate, $R_E$.
- On the payoff date if the rate exceeds $R_E$, the call pays off the difference between the actual rate and $R_E$, times a notional principal times the fraction of the year specified in the contract.
- If the rate is less than $R_E$, the interest rate call expires worthless

NPTEL

# Interest Rate Call

Example:

Given an interest rate call with a designated rate of LIBOR

$R_E$ = 7%, NP = $10 million

Time period of 180 days

Day count convention of 180/360

If the LIBOR rate were 8%, the buyer would receive a pay-off on the pay-off date:

(0.08 −0.07)(180/360)($10000000) = $50000

NPTEL

# Interest Rate Call

- Interest rate call options are often written by commercial banks in conjunction with futures loans they plan to provide to their customers.
- The exercise rate on the option usually is set near the current spot rate, with that rate often being tied to the LIBOR.

## Example

- A company planning to finance a future $5 million inventory 60 days from the present by borrowing from a bank at a rate equal to the LIBOR + 50 basis points at the start of the loan could buy from the bank an interest rate call option with an exercise rate equal to say 7%, expiration of 60 days, and notional principal of $5 million.
- At expiation (60 days later) the company would be entitled to a payoff if rates were higher than 7%.
- If the rate on the loan were higher than 7%, the company would receive a payoff that would offset the higher interest on the loan.

NPTEL

# Interest Rate Put

- An interest rate put, also called a floorlet, gives the buyer a payoff on a specified payoff date if a designated interest rate is below the exercise rate, $R_E$.
- On the payoff date if the rate is less than $R_E$, the put pays off the difference between $R_E$ and the actual rate times a notional principal, times the fraction of the year specified in the contract
- If the rate is more than $R_E$, the interest rate put expires worthless

***Example***

Given an interest rate put with a designated rate of LIBOR, $R_E$ = 7%, NP = $10 million, time period of 180 days, and day-count of 180/360, If the LIBOR rate were 6%, the buyer would receive a pay-off on the pay-off date:

$$(0.07 - 0.06)(180/360)(\$10000000) = \$50000$$

NPTEL

# Interest Rate Put

- A financial or non-financial corporation that is planning to make an investment at some future date could hedge that investment against interest rate decreases by purchasing an interest rate put from a commercial bank, investment banking firm, or dealer.
- Suppose a company was expecting a net cash inflow of $8 million in 60 days from its operations and was planning to invest the funds in a 90-day bank CD paying the LIBOR.
- To hedge against any interest rate decreases, the company could purchase an interest rate put (corresponding to the bank's CD it plans to buy) from the bank with the put having an exercise rate of say 6%, expiration of 60 days, and notional principal of $8 million.
- The interest rate put would provide a payoff for the company if the LIBOR were less than 6%, giving the company a hedge against interest rate decreases.

NPTEL

# Cap

- A plain-vanilla cap is a series of European interest rate call options – a portfolio of caplets.

- For example, a 6%, two-year cap on a three-month LIBOR, with a NP of $10 million, provides, for the next two years, a payoff every three months of (LIBOR − 0.06)(0.25)($10000000) if the LIBOR on the reset date exceeds 6% and nothing if the LIBOR equals or is less than 6%.

- Caps are often written by financial institutions in conjunction with a floating-rate loan and are used by buyers as a hedge against interest rate risk.

NPTEL

## Cap Example

- A company with a floating-rate loan tied to the LIBOR could lock in a maximum rate on the loan by buying a cap corresponding to its loan.
- At each reset date, the company would receive a payoff from the caplet if the LIBOR exceeded the cap rate, offsetting the higher interest paid on the floating-rate loan
- If rates decrease, the company would pay a lower rate on its loan while its losses on the caplet would be limited to the cost of the option.
- With a cap, the company would be able to lock in a maximum rate each quarter and still benefit with lower interest costs if rates decrease.

NPTEL

# Floor

- A plain-vanilla floor is a series of European interest rate put options – a portfolio of floorlets.
- For example, a 8%, two-year floor on a three-month LIBOR, with a NP of $10 million, provides, for the next two years, a payoff every three months of (0.08 – LIBOR)(0.25)($10,000,000) if the LIBOR on the reset date is less than 8% and nothing if the LIBOR equals or exceeds 8%.
- Floors are often purchased by investors as a tool to hedge their floating-rate investments against interest rate declines.
- With a floor, an investor with a floating-rate security is able to lock in a minimum rate each period, while still benefiting with higher yields if rates increase.

NPTEL

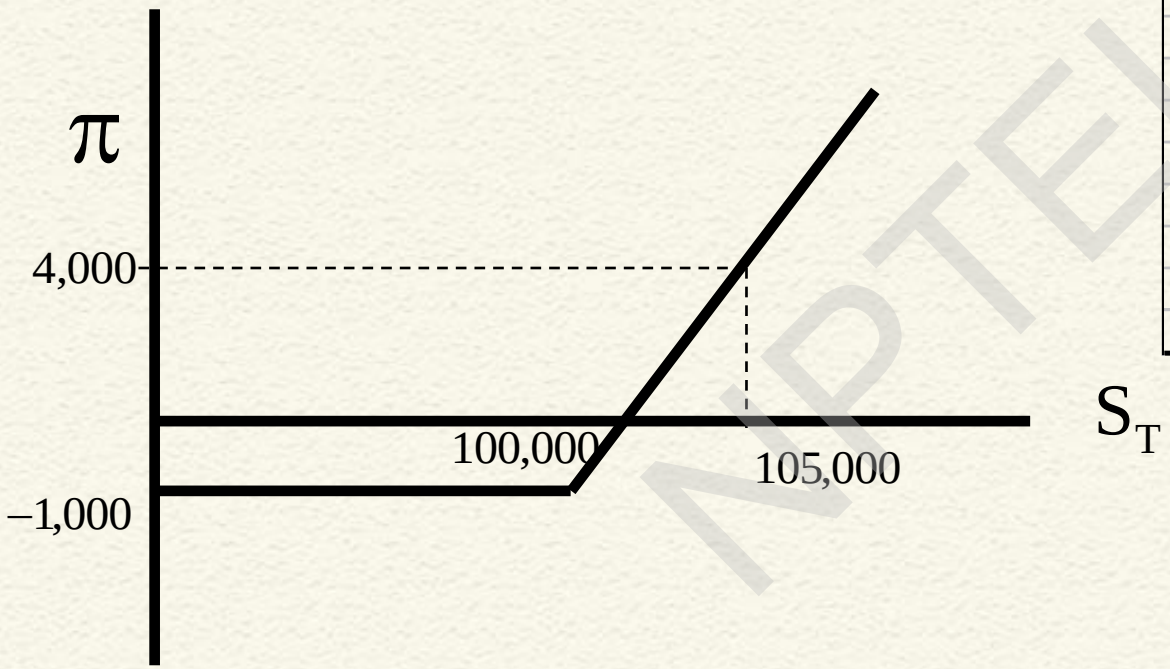
# Spot Option Positions

- Option Strategies can be evaluated in terms of a profit graph. A ***profit graph*** is a plot of the option position's profit, $\pi$, and underlying spot price, S, or futures price, f, relation at expiration or when the option is exercised.
- Consider spot call and put options on a 6% T-bond with Face value of $100000, Maturity at the option's expiration of 10 year, No accrued interest at the option's expiration date and Selling at par. Suppose the T-bond's exercise prices for both call and put options (X) are $100000 (quoted at 100) and an investor can buy the options at a call or put premium of $1,000 (quoted at 1).

NPTEL

# Call Purchase

Buy T-bond call: Exercise Price = $100,000, Call Price = $1,000

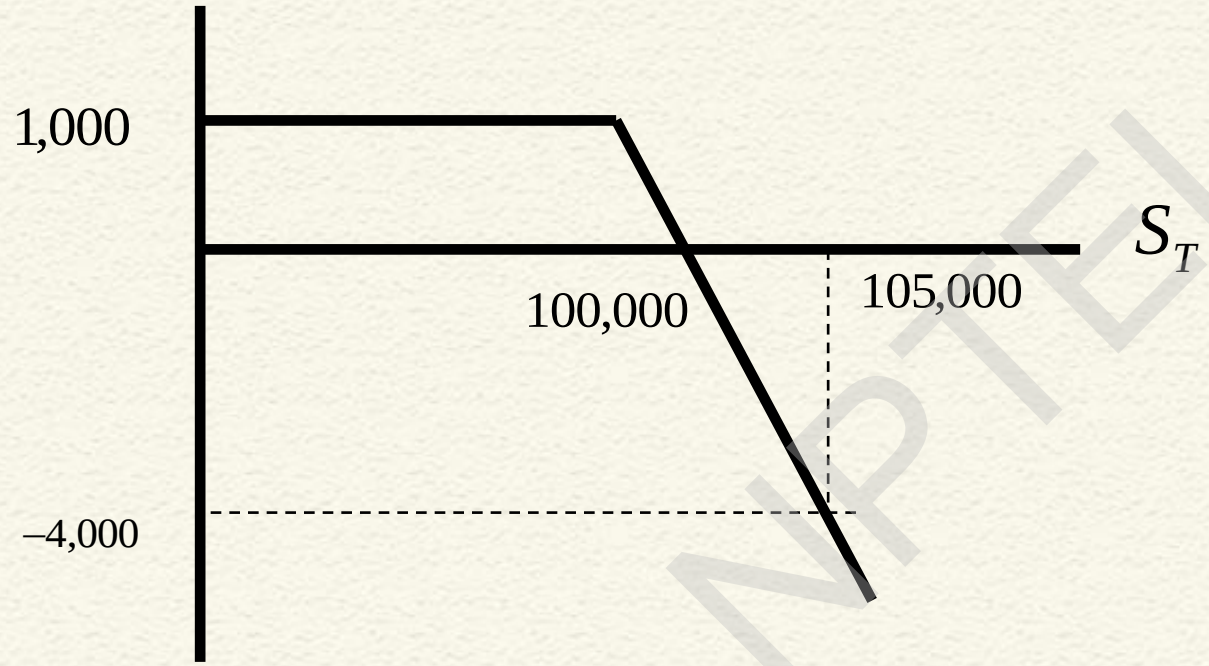



| Call Purchase | |
|---|---|
| Spot Price at T | Profit/Loss |
| 90000 | -1000 |
| 95000 | -1000 |
| 100000 | -1000 |
| 101000 | 0 |
| 102000 | 1000 |
| 103000 | 2000 |
| 104000 | 3000 |
| 105000 | 4000 |
| 106000 | 5000 |

NPTEL

# Call Write

Sell T-bond call for: Exercise Price= 100,000, C=1,000

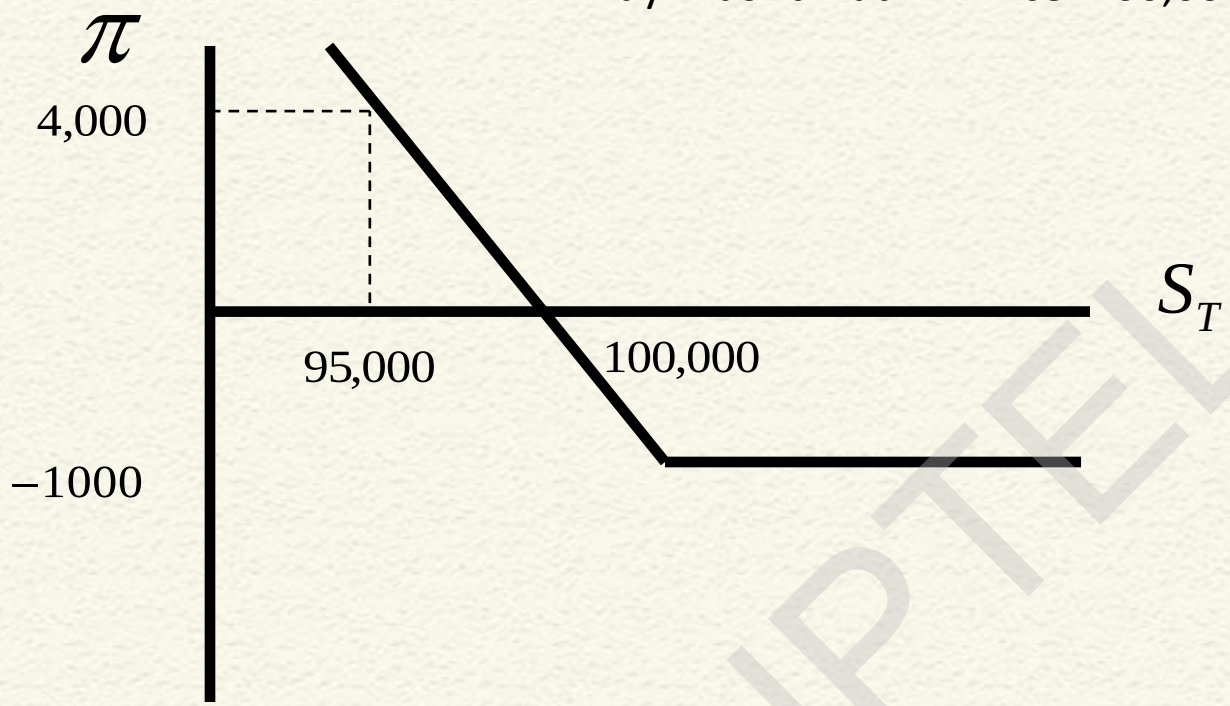



| Naked Call Write | |
|---|---|
| Spot Price at T | Profit/Loss |
| 90000 | 1000 |
| 95000 | 1000 |
| 100000 | 1000 |
| 101000 | 0 |
| 102000 | -1000 |
| 103000 | -2000 |
| 104000 | -3000 |
| 105000 | -4000 |
| 106000 | -5000 |

NPTEL

# Put Purchase

Buy T-bond Put: Ex Price=100,000, P = 1,000

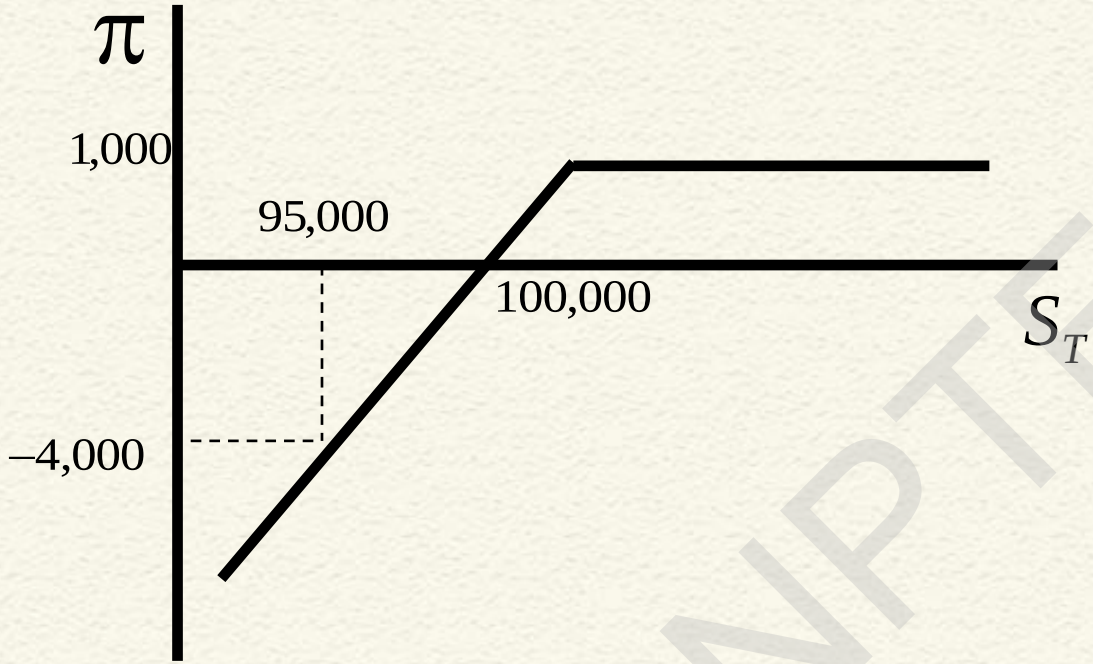



| Put Purchase | |
|---|---|
| Spot Price at T | Profit/Loss |
| 90000 | 9000 |
| 95000 | 4000 |
| 100000 | -1000 |
| 101000 | -1000 |
| 102000 | -1000 |
| 103000 | -1000 |
| 104000 | -1000 |
| 105000 | -1000 |
| 106000 | -1000 |

NPTEL

# Put Write

Sell T-bond put: Exercise price =100,000, P = 1,000




| Naked Put Write | |
|---|---|
| Spot Price at T | Profit/Loss |
| 90000 | -9000 |
| 95000 | -4000 |
| 100000 | 1000 |
| 101000 | 1000 |
| 102000 | 1000 |
| 103000 | 1000 |
| 104000 | 1000 |
| 105000 | 1000 |
| 106000 | 1000 |

NPTEL

# CONCLUSIONS

- An option is a security that gives the holder the right (but not the obligation) to buy (sell) an asset at a specific price (exercise price) on or possibly before a specific date (expiration date)
- A futures option gives the holder the right to take a position in a futures contract
- The premiums on the call and put option contracts on the T-bond and T-note futures are quoted as a percentage of the face value of the underlying bond or note
- In addition to option contracts on specific securities, the OTC market also offers a number of interest-rate option products like interest rate call and the interest rate put
- Option Strategies can be evaluated in terms of a profit graph

NPTEL

# REFERENCES


- Johnson, S. R (2010): Bond Evaluation, Selection and Management, John Wiley & Sons, 2nd Edition.
- Fabozzi, J. Frank and Mann, V. Steven (2005): The Hand Book of Fixed Income Securities, Tata McGraw-Hill, 7th Edition.
- Hull, C. John and Basu Sankarshan (2018): Options, Futures and Other Derivatives, Pearson Education, 10th Edition.


NPTEL




NPTEL ONLINE CERTIFICATION COURSES

# Management of Fixed Income Securities

Prof. Jitendra Mahakud
Department of Humanities and Social Sciences
Indian Institute of Technology Kharagpur

## Lecture 58: Fixed Income Securities Derivatives-III

# CONCEPTS COVERED

- Hedging fixed income positions
- Speculating with interest rate derivatives

NPTEL

# KEYWORDS

- Long hedge
- Short hedge
- Cross hedge
- Maturity gap
- Price sensitivity model
- Intra-commodity spread
- Inter-commodity spread

NPTEL

# Long Hedge - 91-Day T-Bill Future

- A fund manager is expecting a Rs.5 million cash inflow in March that she/ he is planning to invest in T-bills for 91 days. The manager can lock in the yield on the T-bill investment by going long in March T-bill futures contracts.
- Suppose the March T-bill contract is trading at the index price of 95. What should the be the yield ($YTM_f$) on a 91-day investment made at the futures' expiration date in March?

$$f_0(March) = \frac{100 - (5)\left(\frac{90}{360}\right)}{100}(Rs.1M) = Rs.987500$$

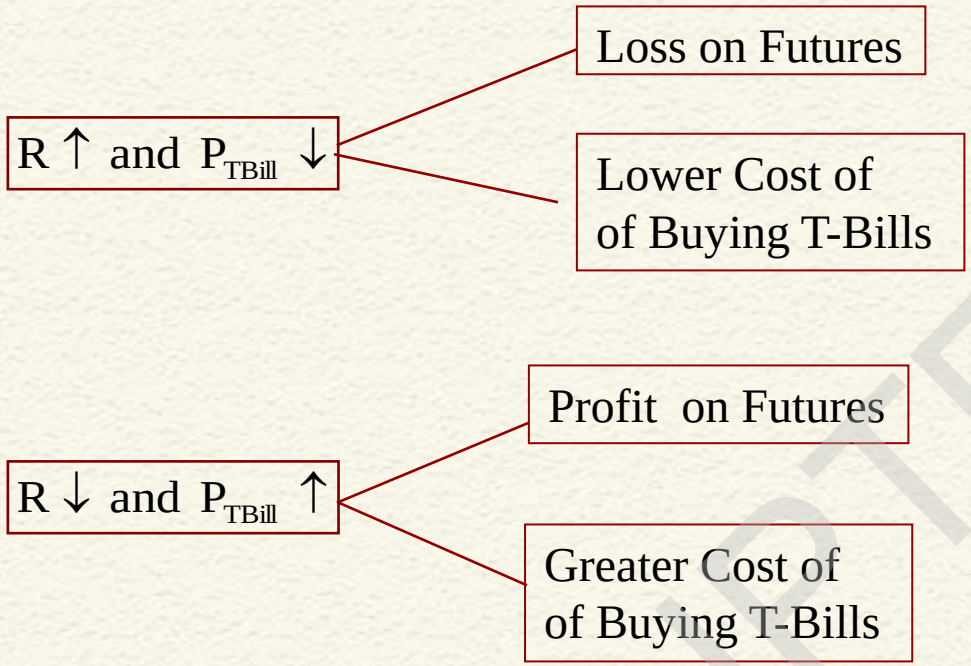
$$YTM_f = \left[\frac{Rs.1M}{Rs.987500}\right]^{365/91} - 1 = 0.051748$$

How many T-Bill futures contract should the fund manager buy to hedge the 91-day investment in order to obtain the 5.1748% yield?

$$n_f = \frac{\text{Investment in March}}{f_0} = \frac{\text{Rs. 5000000}}{\text{Rs. 987500}} = 5.063291 \text{ Long Contracts}$$

NPTEL

# Long Hedge - 91-Day T-Bill Future




NPTEL

# Long Hedge - 91-Day T-Bill Future Investment

Suppose at the March expiration, the spot 91-day T-bill rate is at 4.5%.

$At\ March\ Date, Yield\ on\ T-Bill = 4.5\%$:

$$S_T^{91} = \frac{\text{Rs. } 1{,}000{,}000}{(1.045)^{91/365}} = Rs. 989{,}086$$

$$\pi_f = [Rs. 989{,}086 - Rs. 987{,}500]5.063291 = Rs. 8{,}030.38$$

$$n_{TB} = \frac{Rs. 5{,}000{,}000 + Rs. 8{,}030.38}{Rs. 989{,}086} = 5.063291$$

$$Rate = \left[\frac{5.063291(Rs. 1{,}000{,}000)}{Rs. 5M}\right]^{365/91} - 1 = 0.051748$$

NPTEL

# Long Hedge - 91-Day T-Bill Future Investment

Suppose at the March expiration, the spot 91-day T-bill rate is at 5.5%.

$$S_T^{91} = \frac{Rs.1M}{(1.055)^{91/365}} = Rs.986{,}740$$

$$\pi_f = [Rs.986{,}740 - Rs.987{,}500]5.063291 = -Rs.3{,}848$$

$$n_{TB} = \frac{Rs.5{,}000{,}000 - Rs.3{,}848}{Rs.986{,}740} = 5.063291$$

$$Rate = \left[\frac{5.063291(Rs.1M)}{Rs.5M}\right]^{365/91} - 1 = 0.051748$$

The hedge rate of 5.1748% occurs for any rate scenario.

NPTEL

# Hedging 91-Day T-Bill Future Investment with Calls

- In the example, the investor locked in the yield on the T-bill investment by going long in March T-bill futures contracts.
- Suppose the investor expected higher short-term rates in March but was still concerned about the possibility of lower rates.
- To be able to gain from the higher rates and yet still hedge against lower rates, the investor could alternatively buy March call options on T-bill futures.

NPTEL

# Hedging Future 91-Day T-Bill Investment with Calls

- Suppose there is a March T-bill futures call with
  1. Exercise price = Rs.987,500 (index = 95, $R_D$ = 5)
  2. Price = Rs.1,000 (quote = 4; C = (4)(Rs.250) = Rs.1,000)
  3. March expiration (on both the underlying futures and futures option) occurring at the same time the Rs.5,000,000 cash inflow is to be received
- To hedge the 91-day investment with this call, the investor would need to buy :

$$n_C = \frac{V_T}{X} = \frac{\text{Rs. } 5{,}000{,}000}{\text{Rs. } 987{,}500} \cong 5 calls$$

$$Cost = n_C C = (5)(\text{Rs. } 1{,}000) = \text{Rs. } 5{,}000$$

NPTEL

# Hedging Future 91-Day T-Bill Investment with Calls

| Spot Discount Rates | Spot Price = Futures Price at T (Rs) | Call Cash Flow | Hedged Investment Funds | Number of Bills | YTM |
|---|---|---|---|---|---|
| $R_D$ % | | | Rs. 5000000 + Col 3 | Col (4)/Col(2) | |
| 3.75 | 990625 | 15625 | 5015625 | 5.06 | 0.0516 |
| 4.00 | 990000 | 12500 | 5012500 | 5.06 | 0.0516 |
| 4.25 | 989375 | 9375 | 5009375 | 5.06 | 0.0516 |
| 4.50 | 988750 | 6250 | 5006250 | 5.06 | 0.0517 |
| 4.75 | 988125 | 3125 | 5003125 | 5.06 | 0.0517 |
| 5.00 | 987500 | 0 | 5000000 | 5.06 | 0.0517 |
| 5.25 | 986875 | 0 | 5000000 | 5.07 | 0.0544 |
| 5.50 | 986250 | 0 | 5000000 | 5.07 | 0.0571 |
| 5.75 | 985625 | 0 | 5000000 | 5.07 | 0.0598 |
| 6.00 | 985000 | 0 | 5000000 | 5.08 | 0.0625 |
| 6.25 | 984375 | 0 | 5000000 | 5.08 | 0.0652 |

Cash Flow = $5[\text{Max}(f_T - \text{Rs.}987{,}500), 0]$

YTM = $[(\text{Number of Bill})(\text{Rs.}1{,}000{,}000)]/\text{Rs.}5{,}000{,}000]^{365/91} - 1$

- Thus, for the cost of the call options, the investor is able to establish a floor by locking in a minimum YTM on the Rs.5 million March investment of approximately 5.16%, with the chance to earn a higher rate if short-term rates increase.

NPTEL

# Hedging Future 91-Day T-Bill Investment with Calls

- If T-bill rates were lower at the March expiration, then the investor would profit from the calls and could use the profit to defray part of the cost of the higher priced T-bills.

- If the spot discount rate on T-bills is 5% or less, the investor would be able to buy 5.06 spot T-bills (assume perfect divisibility) with the Rs.5 million cash inflow and the cash flow from the futures calls, locking in a YTM for the next 91 days of approximately 5.16% on the Rs.5 million investment (this exclude the cost of the calls).

- On the other hand, if T-bill rates are higher, then the investor would benefit from lower spot prices while her losses on the call would be limited to just the Rs.5,000 costs of the calls. For spot discount rates above 5%, the treasurer would be able to buy more T-bills the higher the rates, resulting in higher yields as rates increase.

NPTEL

# Short Hedge: Managing the Maturity Gap

- An important use of short hedges is in minimizing the interest rate risk that financial institutions are exposed to when the maturity of their assets does not equal the maturity of their liabilities – **maturity gap**.

***Example:***

- Consider the case of a bank with a maturity gap problem in which its short-term loan portfolio has an average maturity greater than the maturity of the CDs that it is using to finance its loans.
- Suppose in March, the bank makes loans of Rs.100 million, all with maturities of 180 days.
- To finance the loans, though, suppose the bank's customers prefer 90-day CDs to 180-day CDs, and as a result, the bank sells Rs.100 million worth of 90-day CDs at a rate equal to the current LIBOR of 5%.

NPTEL

# Short Hedge: Managing the Maturity Gap

***Example:***

- Ninety days later (in June) the bank would owe Rs.100,000,000$(1.05)^{90/365}$ = Rs.101,210,311
- To finance this debt, the bank would have to sell Rs.101,210,311 worth of 90-day CDs at the LIBOR at that time.
- In the absence of a hedge, the bank would be subject to interest-rate risk.
  - If short-term rates increase, the bank would have to pay higher interest on its planned June CD sale, lowering its interest spread
  - If rates decrease, the bank's spread would increase

NPTEL

# Short Hedge: Managing the Maturity Gap

***Example:***

- Suppose the bank is fearful of higher rates in June and decides to minimize its exposure to market risk by hedging its Rs.101,210,311 CD sale in June with a June Eurodollar futures contract trading at an index value of 95.
- To hedge the liability, the bank would need to go short in 102.491454 June Eurodollar futures (assume perfect divisibility):

$$f_0(June) = \frac{100 - (5)(90/360)}{100}(\text{Rs. } 1{,}000{,}000) = \text{Rs. } 987{,}500$$

$$n_f = \frac{\text{Rs. } 101{,}210{,}311}{\text{Rs. } 987{,}500} = 102.491454\ Short\ Eurodollar\ Contracts$$

- At a futures price of Rs.987,500, the bank would be able to lock in a rate on its June CDs of 5.23376%.
- With this rate and the 5% rate it pays on its first CDs, the bank would pay 5.117% on its CDs over the 180-day period:

$$YTM_f(June) = \left[\frac{\text{Rs. } 1{,}000{,}000}{\text{Rs. } 987{,}500}\right]^{\frac{365}{90}} - 1 = 0.0523376$$

$$YTM_{180} = \left[(1.05)^{90/365}(1.0523376)^{90/365}\right]^{365/180} - 1 = 0.05117$$

NPTEL

# Short Hedge: Managing the Maturity Gap

- The impact that rates have on the amount of funds needed to be financed and the rate paid on them will exactly offset each other, leaving the bank with a fixed debt amount when the June CDs mature in September.

| | | | |
|---|---|---|---|
| (1) June LIBOR | R | 0.045 | 0.055 |
| (2) June spot and expiring futures price | $S_T = f_T$ = Rs.1,000,000/$(1+R)^{90/365}$ | Rs.989,205 | Rs.986,885 |
| (3) Profit on futures | $\pi_f$ = 102.491454[Rs.987,500 - $f_T$] | -Rs.174,748 | Rs.63,032 |
| (4) Debt on March CD | Rs.100,000,000$(1.05)^{90/365}$ | Rs.101,210,311 | Rs.101,210,311 |
| (5) Total funds to finance | Row (4) – Row (3) | Rs.101,385,059 | Rs.101,147,279 |
| (6) Debt at end of period | [Row (5)]$(1+R)^{90/365}$ | 102,491,433 | 102,491,462 |
| (7) Rate paid for 180-day period | [(Row (6))/Rs.100,000,000]$^{365/180}$ | 5.117% | 5.117% |
| | (Allow for rounding differences) | | |

As shown at a October LIBOR's of 4.5% or 5.5%, the bank's total debt at the end of the 180-day period will be Rs.102.491 million, which equates to a rate of 5.117%.

$$R = \left[\frac{\text{Rs. } 102.4914m}{\text{Rs. } 100M}\right]^{365/180} - 1 = 0.05117$$

NPTEL

# Short Hedge: Managing the Maturity Gap

- When the first CDs mature in June, the bank will issue new 90-day CDs at the prevailing LIBOR to finance the Rs.101,210,311 first CD debt plus (minus) any loss (profit) from closing its June Eurodollar futures position.

- If the LIBOR in October has increased, the bank will have to pay a greater interest on the new CD, but it will realize a profit from its futures contracts, decreasing the amount of funds it needs to finance at the higher rate.

- On the other hand, if the LIBOR is lower, the bank will have lower interest payments on its new CDs, but it will also incur a loss on its futures position and therefore will have more funds that need to be financed at the lower rates.
-
- The impact that rates have on the amount of funds needed to be financed and the rate paid on them will exactly offset each other, leaving the bank with a fixed debt amount when the June CDs mature in September

NPTEL

# Managing the Maturity Gap with a Eurodollar Futures Put

- Instead of hedging its future CD sale with Eurodollar futures, the bank could alternatively buy put options on Eurodollar futures.
- By hedging with puts, the bank would be able to lock in or cap the maximum rate it pays on it June CD.
- Suppose the bank decides to hedge its June CD sale by buying June Eurodollar futures puts with
    - Expirations coinciding with the maturity of its June CD
    - Exercise price of 95 (X = Rs. 987,500)
    - Premium of 2 (multiplier = Rs. 250)
- With the June debt from the March CD of Rs.101,210,311, the bank would need to buy 102.491454 June Eurodollar futures puts (assume perfect divisibility) at a total cost of Rs.51,246 to cap the rate it pays on its June CD:

$$n_P = \frac{\text{Rs. } 101{,}210{,}311}{\text{Rs. } 987{,}500} = 102.491454 \; contracts$$

$$Cost = (102.49154)(2)(\text{Rs. } 250) = \text{Rs. } 51{,}246$$

NPTEL

# Managing the Maturity Gap with a Eurodollar Futures Put

| LIBOR% | Spot and Futures Price | Put Cash Flow | June Debt on March CD | June Funds Needed | September Debt Obligation | March to September Hedged Rate |
|---|---|---|---|---|---|---|
| | | | | Col (4) -Col (3) | [Col (5)](1+LIBOR)$^{(90/365)}$ | [Col (6)/$100M]$^{(365/180)}$ - 1 |
| 3.50 | 991553.33 | 0 | 101210311 | 101210311 | 102072483 | 0.042 |
| 3.75 | 990963.66 | 0 | 101210311 | 101210311 | 102133222 | 0.044 |
| 4.00 | 990375.75 | 0 | 101210311 | 101210311 | 102193850 | 0.045 |
| 4.25 | 989789.60 | 0 | 101210311 | 101210311 | 102254368 | 0.046 |
| 4.50 | 989205.21 | 0 | 101210311 | 101210311 | 102314778 | 0.047 |
| 4.75 | 988622.55 | 0 | 101210311 | 101210311 | 102375078 | 0.049 |
| 5.00 | 988041.63 | 0 | 101210311 | 101210311 | 102435270 | 0.050 |
| 5.25 | 987462.42 | 3,852 | 101210311 | 101206459 | 102491454 | 0.051 |
| 5.50 | 986884.93 | 63,040 | 101210311 | 101147271 | 102491454 | 0.051 |
| 5.75 | 986309.14 | 1,22,053 | 101210311 | 101088258 | 102491454 | 0.051 |
| 6.00 | 985735.05 | 1,80,893 | 101210311 | 101029418 | 102491454 | 0.051 |
| 6.25 | 985162.64 | 2,39,560 | 101210311 | 100970751 | 102491454 | 0.051 |
| 6.50 | 984591.91 | 2,98,055 | 101210311 | 100912256 | 102491454 | 0.051 |

NPTEL

# Managing the Maturity Gap with a Eurodollar Futures Put

- If the LIBOR at the June expiration is greater than 5%, the bank will have to pay a higher rate on its June CD, but it will profit from its Eurodollar futures put position, with the put profits being greater, the higher the rate.
- The put profit would serve to reduce part of the Rs.101,210,311 funds the bank would need to pay on its maturing March CD.
- This reduction would, in turn, offset the higher rate it would have to pay on its June CD.
- If the LIBOR is at discount yield of 5% or higher, then the bank would be able to lock in a debt obligation 90 days later of Rs.102,491,451 for an effective 180-day rate of 5.1% (this excludes the cost of the puts).

NPTEL

# Managing the Maturity Gap with a Eurodollar Futures Put

- On the other hand, if the rate is less than or equal to 5%, then the bank would be able to finance its Rs.101,210,311 debt at lower rates.

- For lower rates the bank would realize a lower debt obligation 90 days later and therefore a lower rate paid over the 180-day period.

- For the cost of the puts, hedging the maturity gap with puts allows the bank to lock in a maximum rate on its debt obligation, with the possibility of paying lower rates if interest rates decrease.

NPTEL

# Cross Hedge: Price-Sensitivity Model

- Cross Hedging is hedging a position with a derivative contract in which the asset underlying the derivative is different than the asset to be hedged.

- One model used for cross hedging is the ***price-sensitivity model*** developed by Kolb and Chiang (1981) and Toers and Jacobs (1986).

- This model has been shown to be relatively effective in reducing the variability of debt positions.

- The model determines the number of futures or option contracts that will make the value of a portfolio consisting of a fixed-income security and an interest rate futures contract invariant to small changes in interest rates.

NPTEL

# Cross Hedge: Price-Sensitivity Model

$$n_f = \frac{Dur_S}{Dur_f} \frac{V_0}{f_0} \frac{(1+YTM_f)^T}{(1+YTM_S)^T}$$

where:

- $Dur_S$ = duration of the bond being hedged
- $Dur_f$ = duration of the bond underlying the futures contract (for T-bond futures this would be the cheapest-to-deliver bond)
- $V_0$ = current value of bond to be hedged
- $YTM_S$ = yield to maturity on the bond being hedged
- $YTM_f$ = yield to maturity implied on the futures contract

For option hedging, the number of options (calls for hedging long positions and puts for short positions) using the price-sensitivity model is:

$$n_{options} = \frac{Dur_S}{Dur_{option}} \frac{V_0}{X} \frac{(1+YTM_{option})^T}{(1+YTM_S)^T}$$

$Dur_{option}$ = duration of the bond underlying the option contract

NPTEL

# Cross Hedge: Price-Sensitivity Model (Example: Hedging a Bond Portfolio with T-Bond Futures Puts

- Suppose a bond portfolio manager is planning to liquidate part of his portfolio in June.
- The portfolio he plans to sell consists of investment grade bonds with (i) Weighted Average Maturity of 15.25 Years, (ii) Face Value of Rs.10 Million, (iii) Weighted Average Yield of 8% , (iv) Portfolio Duration of 10 , (v) Current Value of Rs.10 Million
- Suppose the manager would like to benefit from lower long-term rates that he expects to occur in the future but would also like to protect the portfolio sale against the possibility of a rate increase.
- To achieve this dual objective, the manager could buy an July spot or exchange-traded futures put on a T-bond.
- Suppose there is a June 95 (X = Rs.95,000) T-bond futures put option trading at Rs.1156 with the cheapest-to-deliver T-bond on the put's underlying futures being a bond with a current maturity of 15.25 years, duration of 9.818, and currently priced to yield 6.0%.

NPTEL

# Cross Hedge: Price-Sensitivity Model

- Using the price-sensitivity model, the manager would need to buy 81 puts at a cost of Rs.93,636 to hedge his bond portfolio:

$$n_p = \frac{Dur_S}{Dur_p}\frac{V_0}{X}\frac{(1+YTM_p)^T}{(1+YTM_S)^T}$$

$$n_p = \frac{10}{9.818}\frac{\text{Rs. } 10M}{\text{Rs. } 95{,}000}\frac{(1.06)^{15.25}}{(1.08)^{15.25}} \cong 81$$

$$Cost = (81)(\text{Rs. } 1{,}156) = \text{Rs. } 93{,}636$$

NPTEL

# Cross Hedge: Price-Sensitivity Model

- Suppose that in June long-term rates were higher, causing the value of the bond portfolio to decrease from Rs.10 million to Rs.9.1 million and the price on June T-bond futures contracts to decrease from 95 to 86. In this case, the bond portfolio's Rs.900,000 loss in value would be partially offset by a Rs.635,364 profit on the T-bond futures puts:

$$\pi = 81(\text{Rs.}95{,}000 - \text{Rs.}86{,}000) - \text{Rs.}93{,}636 = \text{Rs.}635{,}364$$

- The manager's hedged portfolio value would therefore be Rs.9,735,364; a loss of 2.6% in value (this loss includes the cost of the puts) compared to a 9% loss in value if the portfolio were not hedged.
- On the other hand, if rates in June were lower, causing the value of the bond portfolio to increase from Rs.10 million to Rs.10.5 million and the prices on the June T-bond futures contracts to increase from 95 to 100, then the puts would be out of the money and the loss would be limited to the Rs.93,636 costs of the put options.
- In this case, the hedged portfolio value would be Rs.10,406,365 – a 4.06% gain in value compared to the 5% gain for an unhedged position.

NPTEL

# Speculating with Interest-Rate Derivatives

- Interest rate derivatives are frequently used to speculate on expected interest rate changes.
- A long futures or call position can be taken when interest rates are expected to fall.
- A short futures or put position can be taken when rates are expected to rise.
- Speculating on interest rate changes by taking such **outright** or **naked** futures positions represents an alternative to buying or short selling a bond on the spot market.
- Because of the risk inherent in such **outright futures positions**, though, some speculators form spreads instead of taking a naked position.
- **Intra-commodity Spread:** Long and short in futures on the same underlying asset but with different expirations.
- **Inter-commodity Spread**: Long and short in futures with different underlying assets but the same expiration.

NPTEL

# Intra-Commodity Spread

- More distant futures contracts ($T_2$) are more price-sensitive to changes in the spot price than near-term futures contracts ($T_1$)
- A speculator who expected the interest rate on long-term bonds to *decrease* in the future could form an intra-commodity spread by going: Long in a longer term T-bond futures contract and Short in a shorter term T-bond futures contract
- This type of spread will be profitable if the expectation of long-term rates decreasing occurs.

NPTEL

## Intra-Commodity Spread

- An increase in the price of a T-bond resulting from a decrease in long-term rates, will cause the price on the longer term T-bond futures to increase more than the shorter term one. So, a speculator's gains from his long position in the longer-term futures will exceed his losses from his short position.
- If rates rise, losses will occur on the long position; these losses will be offset partially by profits realized from the short position on the longer term contract
- If a bond speculator believed rates would *increase* but did not want to assume the risk inherent in an outright short position, he could form a spread by going Short in a longer term futures contract and Long in a shorter term futures contract

NPTEL

# Inter-Commodity Spread

- Inter-commodity spreads consist of long and short positions on futures contracts with the same expirations, but with different underlying assets.

- Such spreads can be used to form two active bond strategies:
  - Rate-Anticipation Swap
  - Quality Swap

NPTEL

# CONCLUSIONS

- Fixed income derivatives are extensively used for hedging the risk by the market participants
- The positions are always based on the objective and expectation of the investor
- Interest rate derivatives are also used to speculate on expected interest rate changes

NPTEL

# REFERENCES


- Johnson, S. R (2010): Bond Evaluation, Selection and Management, John Wiley & Sons, 2nd Edition.
- Fabozzi, J. Frank and Mann, V. Steven (2005): The Hand Book of Fixed Income Securities, Tata McGraw-Hill, 7th Edition.
- Hull, C. John and Basu Sankarshan (2018): Options, Futures and Other Derivatives, Pearson Education, 10th Edition.


NPTEL




NPTEL ONLINE CERTIFICATION COURSES

# Management of Fixed Income Securities

Prof. Jitendra Mahakud
Department of Humanities and Social Sciences
Indian Institute of Technology Kharagpur

## Lecture 59: Fixed Income Securities Derivatives-IV

# CONCEPTS COVERED

- Interest rate swap
- Swap valuation
- Forward swap
- Swaption

NPTEL

# KEYWORDS

- Swap Rate
- Swap spread
- Par value swap
- Basis Swap
- Off-market swap

NPTEL

# Interest Rate Swap

- A swap is an agreement to exchange cash flows at specified future times according to certain specified rules.
- **Interest Rate Swaps**: Exchange of fixed-rate payments for floating-rate payments
- The most popular (plain vanilla) interest rate swap is one where LIBOR is exchanged for a fixed rate of interest.
- In an interest rate swap, one company agrees to pay to another company cash flows equal to interest at a predetermined fixed rate on a notional principal for a predetermined number of years. In return, it receives interest at a floating rate on the same notional principal for the same period of time from the other company.
- The floating rate in most interest rate swap agreements is the London Interbank Offered Rate (LIBOR).

NPTEL

# Interest Rate Swap Example

- Consider a hypothetical 3-year swap initiated on March 15, 2016, between Company X and Company Y.
- Suppose X agrees to pay Y an interest rate of 5% per annum on a principal of Rs.100 million, and in return Y agrees to pay X the 6-month LIBOR rate on the same principal.
- X is the fixed-rate payer; Y is the floating rate payer.
- We assume the agreement specifies that payments are to be exchanged every 6 months and that the 5% interest rate is quoted with semi-annual compounding.

NPTEL

# Example: Cash Flows to Company X

| | | ---------Millions of Rupees--------- | | |
|---|---|---|---|---|
| Date | LIBOR Rate | *FLOATING* Cash Flow | *FIXED* Cash Flow | Net Cash Flow |
| Mar.15, 2016 | 4.2% | | | |
| Sept. 15, 2016 | 4.8% | +2.10 | –2.50 | –0.40 |
| Mar.15, 2017 | 5.3% | +2.40 | –2.50 | –0.10 |
| Sept. 15, 2017 | 5.5% | +2.65 | –2.50 | +0.15 |
| Mar.15, 2018 | 5.6% | +2.75 | –2.50 | +0.25 |
| Sept. 15, 2018 | 5.9% | +2.80 | –2.50 | +0.30 |
| Mar.15, 2019 | 6.4% | +2.95 | –2.50 | +0.45 |

- The principal itself is not exchanged. For this reason it is termed the notional principal, or just the notional.
- If the notional principal were exchanged at the end of the life of the swap, the nature of the deal would not be changed in any way.
- The notional principal is the same for both the fixed and floating payments. Exchanging Rs. 100 million for Rs. 100 million at the end of the life of the swap is a transaction that would have no financial value to either company X or Y.

NPTEL

# Interest Rate Swap Example

- Converting a liability from (i) fixed rate to floating rate (ii) floating rate to fixed rate
- Converting an investment from (i) fixed rate to floating rate and (ii) floating rate to fixed rate
- Using a swap to transform a liability
  - Company X has borrowed Rs. 100 million at LIBOR+0.1%, wants to transform a floating rate loan into a fixed rate loan. After it has entered into a swap:
    - Loan payment: LIBOR+0.1%
    - Add:Paid under swap + 5%
    - Less:Received under swap - LIBOR
    - Net Payment 5.1%
- For Y swap is used to transform fixed rate into floating rate. Suppose it has 3-year Rs.100 million loan on which it pays 5.2%
  - Loan payment 5.2%
  - Add: Paid under swap +LIBOR
  - Less: Received under swap - 5%
  - Net payment LIBOR+0.2%

NPTEL

# Interest Rate Swap Example (Using the swap to transform an asset

- Suppose Company X owns Rs.100 million in bonds that will provide 4.7% per annum over the next 3 years. Now company X enters into a swap, wants to switch its assets from fixed to floating rate.
  - Investment income 4.7%
  - Less: Paid under swap -5%
  - Add: Received under swap +LIBOR
  - Net income LIBOR-0.3%
- Company Y is transforming an asset earning floating to fixed. Suppose Y has an investment Rs.100 million that yields LIBOR-0.20. After it has entered into the swap:
  - Investment income LIBOR-0.20
  - Less: Paid under swap - LIBOR
  - Add: Received under swap + 5%
  - Net Investment income 4.8%

NPTEL

# Interest Rate Swap Example

Suppose financial intermediary enters into two offsetting swap transactions with X and Y and receives 0.03% fee per year

| Transform a Liability (Fin int. İnvolved) | | |
|---|---|---|
| | Company X | Company Y |
| Loan payment | LIBOR + 0.1% | 5.2% |
| Add: Paid under swap | + 5.015 | + LIBOR |
| Less: Received under swap | - LIBOR | -4.985% |
| Net Payment (Without fin. int.) | 5.115% (5.1%) | LIBOR+0.215 (LIBOR+ 0.2%) |

| Transform an Asset (Fin int. İnvolved) | | |
|---|---|---|
| | Company X | Company Y |
| In. Income | 4.7% | LIBOR-0.2% |
| Less: Paid under swap | -5.015 | -LIBOR |
| Add: Received under swap | + LIBOR | + 4.985% |
| Net Income (Without a fin. int) | LIBOR-0.315% (LIBOR-0.3%) | 4.785% (4.8%) |

NPTEL

# Interest Rate Swaps' Fundamental Use

- One of the important uses of swaps is in creating a synthetic fixed- or floating-rate liability or asset that yields a better rate than a conventional or direct one
- Synthetic Fixed-Rate Loans and Investments: A **synthetic fixed-rate loan** is formed by combining a floating-rate loan with a fixed-rate payer's position
- Synthetic Floating-Rate Loans and Investments: A **synthetic floating-rate loan** is formed by combining a fixed-rate loan with a floating-rate payer's position.
- Swaps can be viewed as a combination of a fixed-rate bond and flexible-rate note (FRN).
- A **fixed-rate payer position** is equivalent to (i) Buying a FRN paying the LIBOR and (ii) Shorting a fixed-rate bond at the swap's fixed rate.
- A **floating-rate payer position** is equivalent to (i) Shorting a FRN at the LIBOR and Buying a fixed-rate bond at the swap fixed rate

NPTEL

# Swap Rate

- It is the average of (a) the fixed rate that a swap market maker is prepared to pay in exchange for receiving LIBOR (its bid rate) and (b) the fixed rate that it is prepared to receive in return for paying LIBOR (its offer rate).

| Maturity | Bid (%) | Offer (%) | Swap Rate (%) |
|---|---|---|---|
| 2 years | 6.03 | 6.06 | 6.045 |
| 3 years | 6.21 | 6.24 | 6.225 |
| 4 years | 6.35 | 6.39 | 6.370 |
| 5 years | 6.47 | 6.51 | 6.490 |
| 7 years | 6.65 | 6.68 | 6.665 |
| 10 years | 6.83 | 6.87 | 6.850 |

NPTEL

# Swap Market Price Quotes

- **Swap spread**: Swap dealers usually quote two different swap spreads (i) One for deals in which they pay the fixed rate and (ii) One in which they receive the fixed rate
- Swap Spread (Example): 80/86 dealer buys at 80bp over T-note yield and sells at 86 over T-note yield. That is, the dealer will take the fixed payer's position at a fixed rate equal to 80 BP over the T-note yield and take the floating payer's position, receiving 86 bp above the T-note yield.
- The fixed and floating rates are not directly comparable. The T-note assumes a 365-day basis and the LIBOR assumes 360.
- The rates need to be prorated to the actual number of days that have elapsed between settlement dates to determine the actual payments.
- Formula:

$$\text{Fixed} - \text{Rate Settlement Payment}: \quad (\text{Fixed Rate}) \left[ \frac{\text{No. of Days}}{365} \right] \text{NP}$$

$$\text{Floating} - \text{Rate Settlement Payment}: \quad (\text{LIBOR}) \left[ \frac{\text{No. of Days}}{360} \right] \text{NP}$$

NPTEL

# Swap Valuation

- At origination, most plain vanilla swaps have an economic value of zero. This means that neither counterparty is required to pay the other to induce that party into the agreement.

- An economic value of zero requires that the swap's underlying bond positions trade at par – **par value swap**.

- If this were not the case, then one of the counterparties would need to compensate the other. In this case, the economic value of the swap is not zero. Such a swap is referred to as an **off-market swap**.

- Whereas most plain vanilla swaps are originally par value swaps with economic values of zero, as we previously noted, their economic values change over time as rates change.

- That is, existing swaps become off-market swaps as rates change.

NPTEL

# Swap Valuation

In general, the value of an existing swap is equal to the value of replacing the swap – **replacement swap**.

- Formally, the values of the fixed and floating swap positions are:

$$SV^{fix} = \left[\sum_{t=1}^{M} \frac{K^P - K^S}{(1+K^P)^t}\right] NP$$

$$SV^{fl} = \left[\sum_{t=1}^{M} \frac{K^S - K^P}{(1+K^P)^t}\right] NP$$

where:

$K^S$ = Fixed rate on the existing swap
$K^P$ = Fixed rate on current par-value swap
$SV^{fix}$ = Swap value of the fixed position on the existing swap
$SV^{fl}$ = Swap value of the floating position on the existing swap

NPTEL

# Swap Valuation

- These values are obtained by discounting the net cash flows at the current YTM ($K^P$).
- This approach to valuing off-market swaps is often referred to as the **YTM approach**.
- The equilibrium price of a bond is obtained not by discounting the bond's cash flows by a common discount rate, but rather by discounting each of the bond's cash flows by their appropriate spot rates – the rate on a zero-discount bond.
- Valuing bonds by using spot rates instead of a common YTM ensures that there are no arbitrage opportunities from buying bonds and stripping them or buying zero-discount bonds and bundling them.
- The argument for pricing bonds in terms of spot rates also applies to the valuation of off-market swaps.
- Similar to bond valuation, the equilibrium value of a swap is obtained by discounting each of the swap's cash flows by their appropriate spot rates.
- The valuation of swaps using spot rates is referred to as the **zero-coupon approach**.

NPTEL

# Comparative Advantage Argument

AAA Company and BBB Company wish to borrow Rs.10 million for 5 years. AAA wants to borrow floating and BBB wants to borrow fixed

| | *Fixed* | *Floating* |
|---|---|---|
| AAA | 4.0% | 6-month LIBOR − 0.10% |
| BBB | 5.2% | 6-month LIBOR + 0.6% |

BBB has a comparative advantage in floating market and AAA has a comparative advantage in fixed rate market.
They can enter into a swap that AAA ends up with floating rate funds and BBB ends up with fixed rate funds.
Suppose AAA agrees to pay interest at 6 month LIBOR on Rs.10 million and BBB agrees to pay 4.35% per annum on Rs.10 million.

NPTEL

## Comparative Advantage Argument

| | AAA | BBB |
|---|---|---|
| Loan payment | 4% | LIBOR+0.6% |
| Add: Paid under swap | + LIBOR | + 4.35% |
| Less: Received under swap | -4.35% | -LIBOR |
| Net Payment<br>(Before) | LIBOR-0.35%<br>(LIBOR-0.10%)<br>(0.25% gain) | 4.95%<br>(5.2%)<br>(0.25% gain) |

The swap agreement appears to improve the position of both company.
Total gain is: 0.25+0.25=0.50%
If “a” is the difference between the interest rates in fixed market and if “b” is the difference between the interest rates in floating market.
Total gain is; a-b
a = 1.2% (5.2%-5%)
b= 0.7% ( LIBOR+0.6%-(LIBOR-0.1%))
a-b = 1.2-0.7=0.5%

NPTEL

# Forward Swap

- A **forward swap** is an agreement to enter into a swap that starts at a future date at an interest rate agreed upon today.
- Forward swaps provide borrowers and investors with a tool for locking in a future interest rate.
- These can be used to manage interest rate risk for fixed-income positions.
- Financial and non-financial institutions that have future borrowing obligations can lock in a future rate by obtaining forward contracts on fixed-payer swap positions.

NPTEL

# Hedging a Future Loan with a Forward Swap (Examples)

- A company wishing to lock in a rate on a 5-year, fixed-rate Rs.100 million loan to start 2 year from today, could enter a 2-year forward swap agreement to pay the fixed rate on a five-year 8%/LIBOR swap
- At the expiration date on the forward swap, the company could issue Rs.100 million floating-rate debt at LIBOR that, when combined with the fixed position on the swap, would provide the company with a synthetic fixed rate loan paying 8% on the floating debt.
- Alternatively, at the forward swap's expiration date, the company could sell the 5-year 8%/LIBOR swap underlying the forward swap contract and issue 5-year fixed-rate debt.
- If the rate on 5-year fixed rate bonds were higher than 8%, for example at 9%, then the company would be able offset the higher interest by selling its fixed position on the 8%/LIBOR swap to a swap dealer for an amount equal to the present value of a 5-year annuity equal to 1% (difference in rates: 9% − 8%) times the notional principal
- If the rate on 5-year fixed rate loans were lower than 8%, say 7%, then the company would benefit from the lower fixed rate loan, but would lose an amount equal to the present value of a 5-year annuity equal to 1% (difference in rates: 7% − 8%) times the notional principal when it closed the fixed position.

NPTEL

## Swaptions

- A **swaption** is a right, but not an obligation to take a position on a swap at a specific swap rate. A swaption is an option on a swap.

- The purchaser of a swaption buys the right to start an interest rate swap with a specific fixed rate or exercise (strike) rate, with a maturity at or during a specific time period in the future.

- If the holder exercises, he/she takes the swap position, with the swap seller obligated to take the opposite counterparty position.

- For swaptions, the underlying instrument is a forward swap and the option premium is the up-front fee.

NPTEL

## Swaptions

- The swaption can be either a receiver swaption or a payer swaption
- A **receiver swaption** gives the holder the right to receive a specific fixed rate and pay the floating rate: The right to take a floating payer's position
- A **payer swaption** gives the holder the right to pay a specific fixed rate and receive the floating rate :The right to take a fixed payer's position
- Swaptions can be either European or American
- A ***European swaption*** can be exercised only at a specific point in time, usually just before the starting date on the swap.
- An ***American swaption*** is exercisable at any point in time during a specified period of time.

NPTEL

# Swaptions (Speculation)

- Suppose a speculator expects the rate on 5-year fixed rate bonds to increase from their current 8% level.
- Suppose the speculator elects to buy a 1-year European payer swaption on a 5-year, 8%/LIBOR swap with a NP of Rs.10million for 50 bp times the NP: (i) 1 x 5 payer swaption, (ii) Exercise date = 1 year , (iii) Exercise rate = 8%, (iv) Underlying swap = 5-year, 8%/LIBOR with NP = Rs.10 million, (v) Swap position = fixed payer, (vi) Option premium = 50 bp times NP
- On the exercise date, if the fixed rate on a 5-year swap were greater than the exercise rate of 8%, then the speculator would exercise his/her right to pay the fixed rate below the market rate. To realize the gain, the market participant could take 8% fixed-rate payer's swap position obtained from exercising and sell it to another counterparty.
- If the swap rate at the expiration date were less than 8%, then the payer swaption would have no value and the speculator would simple let it expire, losing the premium paid.

NPTEL

# Swaptions (Hedging)

- Swaptions also give investors or borrowers protection against adverse interest rate movements, but still allow them to benefit if rates move in their favor.
- As a hedging tool, swaptions serve as a rate-protection tool:
  - As rates increase, the value of the payer swaptions increases in value, making the payer swaption act as a cap on the rates paid on debt positions.
  - As rates decrease, receiver swaptions increase in value, making them act as a floor on the rates earned from asset positions.

NPTEL

## Cancelable Swap

- A **cancelable swap** is a swap in which one of the counterparties has the option to terminate one or more payments. Cancelable swaps can be callable or putable.
- A **callable swap** is one in which the fixed payer has the right to early termination. If rates decrease, the fixed-rate payer on the swap with this embedded call option to early termination can exercise the right to cancel the swap.
- A **putable swap** is one in which the floating payer has the right to early cancellation. A floating-rate payer with this option may find it advantageous to exercise the early-termination right when rates increase.

NPTEL

# Extendable Swap

- An extendable swap is just the opposite of a cancelable swap.
- It is a swap that has an option to lengthen the terms of the original swap.
- The swap allows the holder to take advantage of current rates and extend the maturity of the swap.
- Like cancelable swaps, extendable swaps can be replicated with a generic swap and a swaption
- The floating payer with an extendable option has the equivalent of a floating position on a generic swap and a receiver swaption.
- The fixed payer with an extendable option has the equivalent of a fixed position on a generic swap and a payer swaption.

NPTEL

# Nongeneric Swaps

- Today, there are a number of nongeneric swaps used by financial and non-financial corporations to manage their varied cash flow and return-risk problems.
- Nongeneric swaps usually differ in terms of their rates, principal, or effective dates.
- For example, instead of defining swaps in terms of the LIBOR, some swaps use the T-bill rate or the prime lending rate.

NPTEL

# Nongeneric Swaps

- ***Non-LIBOR Swap***: Swaps with floating rates different than LIBOR. Example: T-bill rate, CP rate, or Prime Lending Rate.
- ***Delayed-Rate Set Swap*** allows the fixed payer to wait before locking in a fixed swap rate – the opposite of a forward swap.
- ***Zero-Coupon Swap***: Swap in which one or both parties do not exchange payments until maturity on the swap.
- ***Prepaid Swap***: Swap in which the future payments due are discounted to the present and paid at the start.
- ***Delayed-Reset Swap***: The effective date and payment date are the same. The cash flows at time t are determined by the floating rate at time t rather than the rate at time $t - 1$.
- ***Amortizing Swaps***: Swaps in which the NP decreases over time based on a set schedule.

NPTEL

# Nongeneric Swaps

- ***Set-Up Swap or Accreting Swap***: Swaps in which the NP increases over time based on a set schedule
- ***Index Amortizing Swap***: Swap in which the NP is dependent on interest rates.
- ***Equity Swap***: Swap in which one party pays the return on a stock index and the other pays a fixed or floating rate.
- ***Basis Swap***: Swaps in which both rates are floating; each party exchanges different floating payments: One party might exchange payments based on LIBOR and the other based on the T-bill yield.
- ***Total Return Swap***: Returns from one asset are swapped for the returns on another asset.

NPTEL

# CONCLUSIONS

- Interest Rate Swap is the Exchange of fixed-rate payments for floating-rate payments
- Interest swaps are used to convert the nature of liabilities and investments
- The valuation of swaps using spot rates is referred to as the zero-coupon approach
- A forward swap is an agreement to enter into a swap that starts at a future date at an interest rate agreed upon today and forward swaps provide borrowers and investors with a tool for locking in a future interest rate
- A swaption is a right, but not an obligation to take a position on a swap at a specific swap rate

NPTEL

# REFERENCES


- Hull, C. John and Basu Sankarshan (2018): Options, Futures and Other Derivatives, Pearson Education, 10th Edition.
- Johnson, S. R (2010): Bond Evaluation, Selection and Management, John Wiley & Sons, 2nd Edition.
- Fabozzi, J. Frank and Mann, V. Steven (2005): The Hand Book of Fixed Income Securities, Tata McGraw-Hill, 7th Edition.


NPTEL




NPTEL ONLINE CERTIFICATION COURSES

# Management of Fixed Income Securities

Prof. Jitendra Mahakud
Department of Humanities and Social Sciences
Indian Institute of Technology Kharagpur

## Lecture 60: Fixed Income Securities Derivatives-V

# CONCEPTS COVERED

- **Credit default swap**

NPTEL

# KEYWORDS

- CDS spread
- CDS valuation
- Conditional default probability
- Binary CDS
- Basket credit default swap
- CDS forward contract
- CDS option

NPTEL

# Credit Default Swap

- **Credit Default Swaps**: Exchange of premium payments for default protection
- In a standard **credit default swap** (CDS), a counterparty buys protection against default by a particular company or economic entity from a counterparty (seller).
- Buyer of the instrument acquires protection from the seller against a default by a particular company or country (the reference entity)

NPTEL

# Credit Default Swap

- The company or entity is known as the **reference entity** and a default by that entity is known as a **credit event**.
- The **buyer** of the CDS makes periodic payments or a **premium** to the **seller** until the end of the life of the CDS or until the credit event occurs.
- Depending on the contract, if the credit event occurs, the buyer has the right to sell a particular bond (or loan) issued by the company for its par value (physical delivery) or receive a cash settlement based on the difference between the defaulted bond's par value and its market price

NPTEL

# Credit Default Swap (Example)

- Suppose two parties enter into a 5-year CDS with a NP of Rs.200 million.
- The buyer agrees to pay 95 bp annually for protection against default by the reference entity.
- If the reference entity does not default, the buyer does not receive a payoff and ends up paying Rs.1.9 million each year for 5 years.
- If a credit event does occur, the buyer will receive the default payment and pay a final accrual payment on the unpaid premium.
- If the event occurs half way through the year, then the buyer pays the seller Rs.950000.
- If the swap contract calls for physical delivery, the buyer will sell Rs.200 million par value of the defaulted bonds for Rs.200 million.
- If there is a cash settlement, then an agent will poll dealers to determine a mid-market value. If that value were Rs.30 per Rs.100 face value, then the buyer would receive Rs.140 million minus the Rs.950000 accrued interest payment.

NPTEL

# CDS Terms

- In the standard CDS, payments are usually made in arrears either on a quarter, semiannual, or annual basis.
- The par value of the bond or debt is the notional principal used for determining the payments of the buyer.
- In many CDS contracts, a number of bonds or credits can be delivered in the case of a default.
- In the event of a default, the payoff from the CDS is equal to the face value of the bond (or NP) minus the value of the bond just after the default. The value of the bond just after the default expressed as a percentage of the bond's face value is known as the **recovery rate** (RR).

$$\text{CDS Payoff} = (1 - RR)NP - \text{Accrued Payment}$$

$$\text{Payoff} = (1 - 0.30)\text{Rs.}200{,}000{,}000) - \text{Accrued Payment}$$
$$\text{Payoff} = \text{Rs.}140000000 - \text{Accrued Payment}$$

NPTEL

# CDS Terms

- If that value on the Rs.200,000,000 CDS were Rs.30 per Rs.100 face value, then the recovery rate would be 30% and the payoff to the CDS buyer would be Rs.140000000 minus any accrued payment.

$$\text{Payoff} = (1 - 0.30)\text{Rs.}200{,}000{,}000) - \text{Accrued Payment}$$
$$\text{Payoff} = \text{Rs.}140000000 - \text{Accrued Payment}$$

- The payments on a CDS are quoted as an annual percentage of the NP. The payment is referred to as the **CDS spread**.
- Swap bankers function as both brokers and dealers in the CDS market. As **dealers**, they will provide bid and ask quotes on a particular credit entry. CDS are used to manage credit risk on fixed income securities

NPTEL

# CDS Uses (Example 1)

- Consider a bond fund manager who just purchased a 5-year BBB corporate bond at a price yielding 8% and wanted to eliminate the credit risk on the bond.
- To eliminate default risk, suppose the manager bought a 5-year CDS on the bond.
- If the spread on the CDS were equal to 2% of the bond's principal, then the purchase of the CDS would have the effect of making the 8% BBB bond a risk-free bond yielding approximately 6%.
- If the bond does not default, then the bond fund manager will receive 6% from owning the bond and the CDS (8% yield on bond – 2% payment on CDS).
- If the bond defaults, then the bond manager would receive 6% from the bond and CDS up to the time of the default and then would receive the face value on the bond from the CDS seller, which the manager can reinvest form the remainder of the 5-year period.
- In this process, the CDS allows the manager to reduce or eliminate the credit risk on the bond.

NPTEL

# CDS Uses (Example 2)

- Suppose a manager holding a portfolio of 5-year U.S. Treasury notes yielding 6% expected the economy to improve and therefore was willing to assume more credit risk in return for a higher return by buying BBB corporate bonds yielding 8%.
- As an alternative to selling his Treasuries and buying the corporate bonds, the manager could sell a CDS.
- If he were to sell a 5-year CDS on the above 5-year BBB bond to a swap bank for the 2% spread, then the manager would be adding 2% to the 6% yield on his Treasuries to obtain an effective yield of 8%.
- With the CDS, the manager would be able to obtain an expected yield equivalent to the BBB bond yield and would also be assuming the same credit risk associated with that bond.

NPTEL

# Equilibrium CDS Spread

- In equilibrium, the payment or spread on a CDS should be approximately equal to the credit spread (also default risk premium) on the CDS's underlying bond or credit
- Example: If the only risk on a 5-year A rated corporate bond yielding 8% were credit risk and the risk-free rate on 5-year investment were 6%, then the bond would be trading in the market with a 2% credit spread.
- If the spread on 5-year CDS on the A rated bond were 2%, then an investor could obtain a 5-year risk-free investment yielding 6% by either Buying a 5-year Treasury or Buying the 5-year A rated corporate yielding 8% and purchasing the CDS on the underlying credit at a 2% spread
- If the spread on a CDS is not equal to the credit spread on the underlying bond, then an arbitrage opportunity would exist by taking positions in the bond, risk-free security, and the CDS.

NPTEL

# Equilibrium CDS Spread

## Let CDS Spread = 1% < Default Spread = 2%

- An **investor** looking for a 5-year risk-free investment would find it advantageous to create the synthetic risk-free investment with the A rated bond and the CDS.
- The investor could earn 1% more than the yield on the Treasury by creating a synthetic treasury by buying the 5-year A rated corporate yielding 8% and purchasing the CDS on the underlying credit at a 1%
- If the swap bank were offering the above CDS for 1% instead of 2%, then an **arbitrager** could also realized a free return equivalent to a 5-year cash flow of 1% of the par value of bond by
    - Shorting the Treasury at 6% (or borrowing at 6%)
    - Using the proceeds to buy the A rated corporate bond
    - Buying the CDS
- These actions by investors and arbitrageurs, in turn, would have the impact of pushing the spread on the CDS towards 2% – the underlying bond's credit risk spread.

NPTEL

# Equilibrium CDS Spread

## Let CDS Spread = 3% > Default Spread = 2%

- If the swap bank were offering the **CDS at a 3% spread**, then an **investor** looking for a 5-year risk-free investment would obviously prefer a 6% Treasury yielding 6% to a synthetic risk-free investment formed with the 5-year A rated corporate bond yielding 8% and a CDS on the credit requiring a payment of 3%.
- A more **aggressive investor** looking to invest in the higher yielding 5-year A rated bonds could earn 1% more than the 8% on the A rated bond by creating a synthetic 5-year A rated bond by (i) Purchasing the 5-year Treasury at 6% and (ii) selling the CDS at 3%
- A **bond portfolio manager** holding 5-year A rated bond yielding 8% could pick up an additional 1% yield with the same credit risk exposure by (i) Selling the A rated bond, (ii) Selling the CDS at 3% (iii) Using the proceeds from the bond sale to buy the 5-year Treasuries yielding 6%

NPTEL

# Equilibrium CDS Spread

## Let CDS Spread = 3% > Default Spread = 2%

- If the swap bank were offering the CDS at a 3% spread, then an **arbitrager** could realized a free return equivalent to a 5-year cash flow of 1% of the par value on the bond by (i) Shorting the A rated bond, (ii) Selling the CDS, (iii) Using proceeds from bond sale to purchase 5-year Treasuries
- With these positions, the **arbitrageur** would receive for each of the next 5 years 6% from her Treasury investment and 3% from the CDS, but only pay 8% on shorting A rated bond position.
- The holdings of Treasury securities would enable the investor to cover the investor's obligation on the CDS if there was a default.
- In the event of a default the investor would be able to pay the CDS holder from the net proceed from selling the Treasuries and closing his/her short A rated bond by buying back the corporate bonds at their defaulted recovery price
- The actions of the investors, bond portfolio managers, and arbitrageur would have the effect of pushing the spread on CDS from 3% to 2%.

NPTEL

# CDS Valuation

- The total value of a CDS's payments is equal to the sum of the **present values of the periodic CDS spread** (Z) times the NP over the life of the CDS, discounted at the risk-free rate (R):

$$\text{PV}(\text{CDS Payments}) = \sum_{t=1}^{M} \frac{\text{Z NP}}{(1+\text{R})^{t}}$$

- The present value of the payment on the 5-year CDS with an equilibrium spread of 2% and a NP of Rs.1 would be:

$$PV(CDS\ Payments) = \sum_{t=1}^{5} \frac{(0.02)(\text{Rs. } 1)}{(1.06)^{t}} = \text{Rs. } 0.084247$$

- The buyer (seller) of this 5-year CDS would therefore be willing to make (receive) payments over five years that have a present value Rs.0.084247.

NPTEL

# CDS Valuation

- As the spread can also be viewed as an expected loss of principal, the present value of the payments is also equal to the expected default protection the buyer (seller) receives (pays).
- The value of the CDS protection is equal to the **present value of the expected payout in the case of default.**

$$PV(\text{Expected Payout}) = \sum_{t=1}^{M} \frac{p_t \, NP(1-RR)}{(1+R)^t}$$

where:

$p_t$ = probability of default in period t conditional on no earlier default (conditional probability)

RR = recovery rate (as a proportion of the face value) on the bond at the time of default

NP = notional principal equal to the par value of the bond.

- Conditional default probabilities are referred to as **default intensities**.
- Over a period of time, these probabilities will change, increasing or decreasing depending on the quality of the credit.

NPTEL

# CDS Valuation

- Instead of defining a CDS's expected payout in terms periodic probability density, $p_t$, the CDS's expected payout can alternatively by defined by the **average conditional default probability, $\bar{p}$ :**

$$PV(Expected\ Payout) = \sum_{t=1}^{M} \frac{\bar{p}\, NP(1-RR)}{(1+R)^t} = \bar{p}NP(1-RR)\sum_{t=1}^{M} \frac{1}{(1+R)^t}$$

- Given the equilibrium spread of 0.02 and assuming a recovery rate of 30% if the underlying bond defaults, the implied probability density for our illustrative CDS would be 0.02857.

- This implied probability is obtained by solving for the that makes the present value of the expected payout equal to present value of the payments of 0.084247.

NPTEL

# CDS Valuation (Example)

$$PV(\text{Expected Payout}) = PV(\text{Payments})$$

$$\sum_{t=1}^{M} \frac{\bar{p}\, NP(1-RR)}{(1+R)^t} = \sum_{t=1}^{M} \frac{Z\, NP}{(1+R)^t}$$

$$\bar{p} = \frac{Z}{(1-RR)}$$

$$\bar{p} = \frac{0.02}{(1-0.30)} = 0.02857$$

$$PV(\text{Expected Payout}) = \sum_{t=1}^{M} \frac{\bar{p}\, NP(1-RR)}{(1+R)^t}$$

$$PV(\text{Expected Payout}) = \sum_{t=1}^{5} \frac{(0.02857)(1)(1-0.30)}{(1.06)^t}$$

$$PV(\text{Expected Payout}) = 0.084247$$

If there were no recovery (RR = 0), then the implied probability would be equal to the spread Z, which as noted can be thought of as the probability of default of principal.

The probability density implied by the market is referred to as the **risk-neutral probability** because it is based on an equilibrium spread that is arbitrage free.

NPTEL

# Other Credit Derivatives

- A **binary CDS** is identical to the generic CDS except that the payoff in the case of a default is a specified amount. The fixed payoff is the principal on the underlying credit.
- The only difference between the generic and binary swap is that the generic CDS adjust the payoff by subtracting the recovery value whereas the binary CDS does not.
- In a **basket credit default swap**, there is a group of reference entities or credits instead of one and there is usually a specified payoff whenever one of the reference entities defaults.
- Similar to a regular CDS except that several reference entities are specified
- In a first to default swap there is a payoff when the first entity defaults. Second, third, and $n$th to default deals are defined similarly

NPTEL

# Other Credit Derivatives

- Basket CDS can vary by the type of agreement governing the swap:
  - **Add-up basket CDS** provides a payout when any reference credit in the basket defaults
  - **First-to-default CDS** provides a payout only when the first entry defaults
  - **Second-to-default CDS** provides a payout when the second default occurs
  - **Nth-to-default CDS** provides a payout when the nth credit entry defaults

NPTEL

# CDS Forward and Option Contracts

- A **CDS forward contract** is a contract to take a buyer's position or a seller's position on a particular CDS at a specified spread at some future date.
- CDS forward contract provide a tool for locking in the credit spread on future credit position.
- A **CDS option** is an option to buy or sell a particular CDS at specified swap rate at a specified future time

NPTEL

# CONCLUSIONS

- In a **credit default swap**, a counterparty buys protection against default by a particular company or economic entity from a counterparty (seller)

- The total value of a CDS's payments is equal to the sum of the **present values of the periodic CDS spread** times the NP over the life of the CDS, discounted at the risk-free rate

- Credit default swaps are used to eliminate the default risk

NPTEL

# REFERENCES


- Johnson, S. R (2010): Bond Evaluation, Selection and Management, John Wiley & Sons, 2nd Edition.
- Hull, C. John and Basu Sankarshan (2018): Options, Futures and Other Derivatives, Pearson Education, 10th Edition.


NPTEL




NPTEL